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HADRONIC FORM FACTORS: PERTURBATIVE QCD vs QCD SUM RULES *

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Abstract

The challenging problem of applicability of the perturbative QCD to exclusive processes is reviewed. The basic ingredients both of the asymptotic large- Q^2 analysis and of the QCD sum rule approach are analyzed for the simplest and most well-studied example of the pion electromagnetic form factor. The main conclusion is that for accessible energies and momentum transfers the soft (nonperturbative) contributions dominate over those due to the hard quark rescattering subprocesses.

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1. INTRODUCTION

Elastic form factors for many years have been a subject of very intensive studies – both experimental and theoretical. The reason is that they contain an important information about the internal structure of the “elementary” particles. In the nonrelativistic quantum mechanics, *e.g.*, the form factor is just the Fourier transform of the charge distribution inside a system. In the (light-cone) quantum field theory, the form factor of a two-body bound state is given by a convolution

$$F(q^2) \sim \int \psi_P(x, k_\perp) \psi_P(x, k_\perp + xq) d^2 k_\perp dx \quad (1)$$

involving initial and final state wave functions. If q , the momentum transfer to the system, is large enough, studying the form factor one can extract information about the high- k_\perp behavior of the bound state wave function. This is especially important in quantum chromodynamics where the wave functions accumulate nonperturbative information about the hadronic structure. In particular, the low- k_\perp part of the wave function dominated by soft interactions – $\psi_P^{soft}(x, k_\perp)$ – cannot be calculated in any perturbative way.

However, a crucial observation made at the end of 70's [1, 2, 3, 4] is that the high- k_\perp tail, (to be referred to as $\psi_P^{hard}(x, k_\perp)$) can be calculated within the perturbative QCD approach:

$$\psi_P^{hard} \sim V \otimes \psi_P^{soft}, \quad (2)$$

with V being the perturbative kernel describing short-distance interactions. The kernel starts with the one-gluon exchange term, but includes also the higher order ones. Displaying ψ as a sum of the soft and hard components

$$\psi = \psi^{soft} + \psi^{hard} \quad (3)$$

and assuming eq. (2), one arrives at the QCD factorization expansion [5]. It states essentially that the pion form factor, *e.g.*, can be represented as a sum of terms of increasing complexity (fig.1).

The first term (purely *soft* contribution, fig.1a) contains no short-distance (SD) subprocesses. For large Q^2 it vanishes like $1/Q^4$ or faster. The second term (fig.1b) contains a *hard* gluon exchange and behaves like $O(\alpha_s/Q^2)$ for large Q^2 . There are also corrections to the hard term: higher order corrections (fig.1c) containing extra α_s factors and higher twist $O((\alpha_s/Q^2)(M^2/Q^2)^n)$ corrections (fig.1d). Thus, the perturbatively calculable one-gluon-exchange diagram asymptotically dominates over all other contributions, and one can apply perturbative QCD to study the large- Q^2 behaviour of hadronic form factors.

It is worth mentioning here that perturbative QCD is now the main theoretical tool to describe hard hadronic processes. Factorization theorems formulated at the end of 70's (and more carefully investigated during the 80's) ensure that, for a sufficiently large momentum transfer Q , one can separate the factors describing the long-distance (nonperturbative) quark-gluon interactions from the amplitudes corresponding to the short-distance subprocesses, the latter being calculable within perturbation theory. This approach is especially successful for

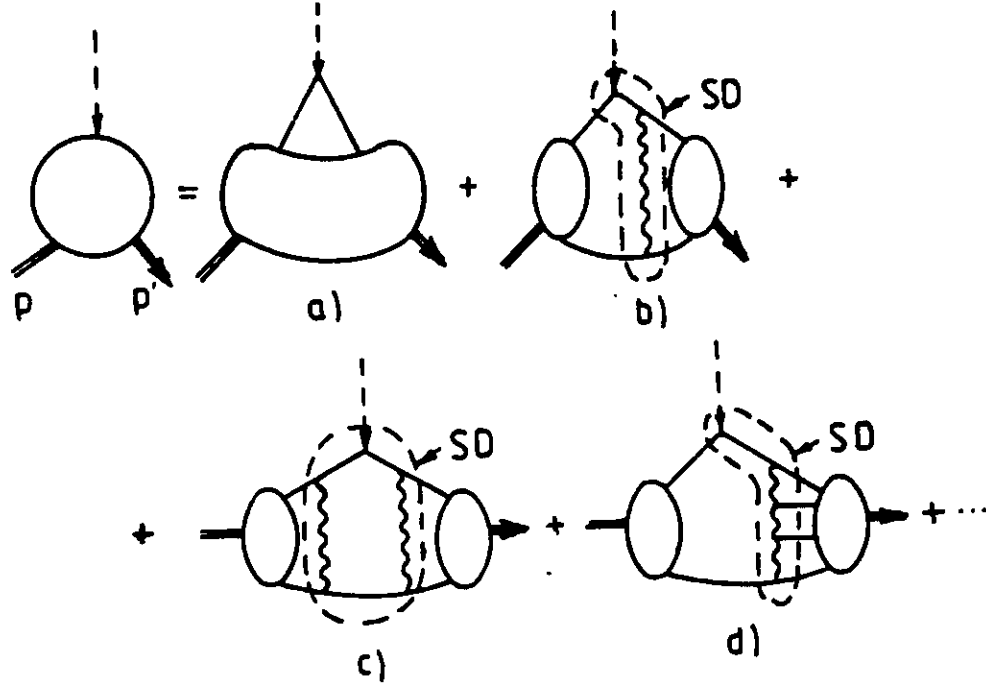


Fig.1 Structure of factorization for the pion form factor

hard inclusive (deeply inelastic) hadronic processes where the QCD parton picture is now the most popular way of describing them.

For exclusive processes, however, the situation is much worse. Though the perturbative QCD is undoubtedly an adequate tool to study the asymptotic $Q^2 \rightarrow \infty$ behavior of the hard exclusive reactions, there is still no agreement about whether the accessible momentum transfers $Q < 6 \text{ GeV}$ are large enough to be treated as the asymptotic ones. In fact, one can find in the literature two opposite viewpoints concerning this problem (compare, *e.g.*, refs. [6, 7] and [8, 9]). My goal in this talk is to discuss the problem of applicability of the perturbative QCD predictions for exclusive processes. I will concentrate on the simplest and most studied example of the pion electromagnetic form factor to demonstrate the basic ingredients of both perturbative and semi-perturbative (within the QCD sum rule method [10]) approaches to exclusive processes in QCD. We will also briefly discuss some more complicated processes.

2. PERTURBATIVE QCD FOR EXCLUSIVE PROCESSES: THE PION FORM FACTOR

The first result of perturbative QCD for exclusive processes is the prediction for the asymptotic behavior of the pion electromagnetic form factor [1, 2, 3, 4]:

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy \frac{2g^2}{3xyQ^2} \varphi(x) \varphi(y), \quad (4)$$

where $\varphi(x)$ is the pion wave function giving the probability amplitude to find the pion in a state where quarks carry fractions xP and $(1-x)P$ of its longitudinal momentum P ; $q = P' - P$ is the momentum transfer to the pion, $q^2 = -Q^2 < 0$; the combination

$$xyQ^2 = -(xP - yP')^2 \equiv -k^2$$

is the virtuality of the exchanged gluon (fig.2) and g is the quark-gluon coupling constant. This formula provides the QCD justification for the quark counting rules [11, 12]

$$F_\pi(Q^2) \sim (1/Q^2)^{n-1} \quad (5)$$

in the pion ($n = 2$) case.

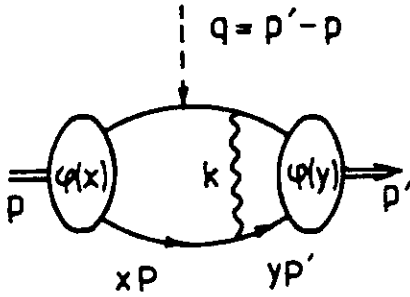


Fig.2 Structure of the leading high- Q^2 contribution to the pion form factor.

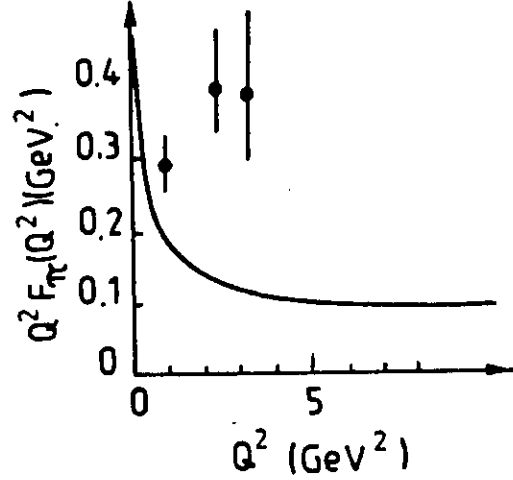


Fig.3 Experimental data on $Q^2 F_\pi(Q^2)$ and the asymptotic QCD prediction.

There are also higher order corrections to the short-distance quark rescattering amplitude. In particular, they induce a logarithmic Q^2 -dependence both of the running QCD coupling constant $\alpha_s \rightarrow \alpha_s(Q^2)$ and of the wave functions: $\varphi(x) \rightarrow \varphi(x, Q^2)$. The latter effect is similar to the scaling violation for the parton distribution functions. In the asymptotic $Q^2 \rightarrow \infty$ limit the pion wave function acquires a very simple and natural form [5, 4, 6]

$$\varphi_\pi(x, Q^2 \rightarrow \infty) \rightarrow \varphi_\pi^{as}(x) = 6f_\pi x(1-x), \quad (6)$$

where $f_\pi = 133 \text{ MeV}$ is the pion decay constant setting the wave function normalization. Thus, using eqs. (4) and (6) one gets the "absolute" prediction for the pion form factor [1, 2, 3, 4, 5, 6]

$$F_\pi^{as}(Q^2) = 8\pi\alpha_s f_\pi^2 / Q^2. \quad (7)$$

However, if one calculates the numerical value of the "almost scaling" combination $Q^2 F_\pi(Q^2)$ taking for α_s the standard low- Q^2 value $\alpha_s \approx 0.3$, one finds that $Q^2 F_\pi^{as}(Q^2) \approx 0.13$ - the result by factor of 3 lower than the experimental [13] one.

The simplest idea then is that α_s is in fact larger. One can even insist that the asymptotic freedom formula

$$\alpha_s(Q^2) = \frac{4\pi}{9 \log(Q^2/\Lambda^2)} \quad (8)$$

should be used. In this case, however, the curve $Q^2 F_\pi^{aa}(Q^2) = 8\pi f_\pi^2 \alpha_s(Q^2)$ is almost orthogonal to the curve $Q^2 F_\pi^{pp}(Q^2)$, the intersection being only one point around $Q^2 = 1 \text{ GeV}^2$ if $\Lambda \sim 100 - 200 \text{ MeV}$ (fig.3).

A popular way (see, e.g., refs.[14, 15]) to avoid the rapid variation of the $\alpha_s(Q^2)$ is to assume that $\alpha_s(Q^2)$ in fact "freezes" for low Q^2 :

$$\alpha_s(Q^2) \rightarrow \alpha_s^{\text{frozen}}(Q^2) = \frac{4\pi}{9 \log((Q^2 + \mu^2)/\Lambda^2)}, \quad (9)$$

with μ being a scale of the order of several hundreds MeV . In this case $\alpha_s(Q^2)$ is almost constant. Note also, that the argument of α_s in our case should be proportional, but not necessarily equal to Q^2 . A more relevant scale is xyQ^2 – the virtuality of the exchanged gluon (see, e.g., [16]). But with $\alpha_s \rightarrow \alpha_s(xyQ^2)$ one gets into much trouble with the region of small x and y unless the effective coupling constant $\alpha_s(Q^2)$ is frozen, and this is another motivation for using eq. (9).

To understand the mechanism of the freezing one should notice that, due to the nonperturbative effects, the gluons inside the loops inducing the Q^2 -dependence of α_s acquire an effective mass m_g in the low momentum region. In other words, the gluonic propagators are modified: $1/l^2 \rightarrow 1/(l^2 - m_g^2)$ and one obtains something like $\log(Q^2 + 4m_g^2)$ instead of $\log(Q^2)$ in eq.(8). Such a modification is very natural in view of the confinement phenomenon: there are no gluons whose wavelength is larger than the confinement radius R_{conf} and this is equivalent to a strong suppression for the propagation of particles with small momenta.

Since $\alpha_s(Q^2)$, in any case, varies slowly only if it is small ($\alpha_s \lesssim 0.3$), one still needs a factor of 3 to describe the data by eq.(4). The only way now is to assume that the pion wave function $\varphi(x, Q^2)$ for low Q^2 strongly differs from its asymptotic form $\varphi^{as}(x)$. Referring to the QCD sum rules for the second and the fourth moments of the pion wave function, Chernyak and A.Zhitnitsky argued that [17]

$$\varphi^{CZ}(x) = 30 f_\pi x(1-x)(1-2x)^2. \quad (10)$$

The use of the double-humped CZ wave function increases the result by factor 25/9 compared to the asymptotic prediction (7).

Summarizing, to describe the data by eq. (4) one should take the CZ form for $\varphi(x)$ and α_s of order of 0.3. To avoid an increase of the combination $Q^2 F_\pi(Q^2)$ for small Q^2 (where experimentally it goes down), one should use a frozen version, eq. (9), for the QCD coupling constant [15]. The same procedure was used [14] for the proton magnetic form factor. The relevant wave functions $\varphi(x_1, x_2, x_3)$, due to Chernyak and I.Zhitnitsky [18] also have humps and are highly asymmetric with respect to x_1, x_2, x_3 . Using them one can get curves for $G_M^p(Q^2)$ close to experimental data.

Still, there is a highly disturbing observation concerning the above "successes" of the asymptotic QCD predictions: the bulk part of the relevant contributions comes from the regions where the virtualities of the exchanged gluons (xyQ^2 in the pion case) are not very large [9]. One can easily verify that, with the CZ wave function, 50% of the whole contribution is due to the regions where both x and y are smaller than 0.2 (xyQ^2 smaller than $Q^2/25$, i.e., smaller than 0.15 GeV^2 for $Q^2 < 4 \text{ GeV}^2$!) and 40% is due to the regions where either x or y is smaller than 0.2. Only 1.5% of the total contribution comes from the region, where both x and y are larger than $1/2$ and one can treat the exchanged gluon as sufficiently virtual to rely on asymptotic freedom. More than 90% is due to the regions of small virtualities where the free-field approximation $D(k^2) \sim 1/k^2$ is more than questionable. As we discussed above, one should expect that for small k^2 the propagator is modified

$$1/k^2 \rightarrow 1/(k^2 - M^2)$$

due to the confinement effects. Now, if one substitutes the denominator factors like xyQ^2 by $(xyQ^2 + M^2)$ with $M \sim 300 - 500 \text{ MeV}$, one immediately observes that the resulting value for $Q^2 F_\pi(Q^2)$ is smaller than the experimental one by a factor of 10, both for the asymptotic and CZ wave functions.

Such a sensitivity of the results to parameters like M (serving as an infrared cut-off) simply means that we are outside the applicability region of perturbative QCD. Furthermore, the basic principle of the whole factorization approach is that any contribution coming from a region where $|k^2| < \mu_0^2 \sim 1 \text{ GeV}^2$ cannot be treated as a short-distance subprocess: it should be considered as a part of a soft (and, generally speaking, nonfactorizable) contribution.

As we discussed above, the pion form factor, according to the factorization theorem [5, 19], is a sum of various terms (fig. 1). The soft contribution (fig. 1a) dominates for small Q^2 . It is this term that provides the normalization condition $F_\pi(Q^2 = 0) = 1$. On the other hand, the gluon exchange diagram dominates as $Q^2 \rightarrow \infty$. The crucial question is: where the asymptotic regime sets in? To begin with, one should take into account that the hard gluon exchange diagram 1b, according to the usual "loop counting" is suppressed by the $\alpha_s/\pi \approx 1/10$ factor. On the other hand, the soft diagram is suppressed, in the large- Q^2 region, by a power of M^2/Q^2 with M being a typical hadronic scale of an order of 1 GeV . Thus, the two terms are of comparable magnitude for

$$Q^2 \sim M^2/(\alpha_s/\pi) \sim 10 M^2 \sim 10 \text{ GeV}^2.$$

The soft contribution seems to be dominant in the whole accessible Q^2 -range, while the hard term contributes only something like 10%!

Applying the same estimate for the nucleons (in that case the asymptotically dominant diagrams have two gluon exchanges) one derives that the soft diagram is more important for

$$Q^2 < M^2/(\alpha_s/\pi)^2 \sim 100 M^2 \sim 100 \text{ GeV}^2,$$

i.e., also up to Q^2 values not accessible experimentally.

Of course, there might be various estimates of the magnitude of "the typical hadronic scale". In particular, Brodsky argued [20] that the relevant scale M for the pion is f_π which

is only 133 MeV and, hence, the soft diagram can be ignored above $Q^2 \sim f_\pi^2/(\alpha_s/\pi) \sim (400 \text{ MeV})^2$. To demonstrate that the scale M is really much larger than f_π (in fact, $M \sim 2\pi f_\pi$) one should be able to calculate the soft contribution in some reliable way.

3. QCD SUM RULES AND HADRONIC FORM FACTORS

Among the existing approaches to the analysis of the nonperturbative effects in QCD the most close to perturbative QCD is the QCD sum rule method [10]. Let us formulate its basic ideas within the context of our problem.

It is evident that one cannot directly study the soft contribution with the on-shell pions, because then only long distances are involved. But perturbative QCD can be applied in a situation when all relevant momenta q, p_1, p_2 are spacelike and sufficiently large: $|q^2|, |p_1^2|, |p_2^2| > 1 \text{ GeV}^2$. To describe the virtual pions one should use some interpolating field, the usual (for the QCD sum rule practitioners) choice being the axial current $j_5^\alpha = \bar{d}\gamma_5\gamma^\alpha u$. Its projection onto the pion state $|P, \pi\rangle$ is proportional to f_π :

$$\langle 0 | j_5^\alpha | P, \pi \rangle = i f_\pi P^\alpha. \quad (11)$$

Via the dispersion relation

$$T(p_1^2, p_2^2, q^2) = \frac{1}{\pi^2} \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} \quad (12)$$

one can relate the amplitude $T(p_1^2, p_2^2, q^2)$ to its time-like counterpart $\rho(s_1, s_2, q^2)$ containing the double pole term

$$\rho_\pi(s_1, s_2, q^2) = \pi^2 f_\pi^2 \delta(s_1 - m_\pi^2) \delta(s_2 - m_\pi^2) F_\pi(Q^2) \quad (13)$$

corresponding to the pion form factor. However, the axial current has nonzero projections onto other hadronic states (A_1 -meson, say) as well, and the spectral density $\rho(s_1, s_2, q^2)$ contains also the part $\rho^{\text{higher states}}(s_1, s_2, q^2)$ related to other elastic and transition form factors. This is the price for our going off the pion mass shell. The problem now is to pick out the F_π term from the whole mess.

Of course, calculating $T(p_1^2, p_2^2, q^2)$ in the lowest orders of perturbation theory one never observes something like the pion pole: one obtains a smooth function $\rho^{\text{pert}}(s_1, s_2, q^2)$ corresponding to transitions between the free-quark $\bar{u}d$ -states with invariant masses s_1 and s_2 , respectively. The difference between "exact" density $\rho(s_1, s_2, q^2)$ and its perturbative analog $\rho^{\text{pert}}(s_1, s_2, q^2)$ is reflected by additional nonperturbative contributions to $T(p_1^2, p_2^2, q^2)$. These contributions are due to quark and gluon condensates $\langle \bar{q}q \rangle$, $\langle GG \rangle$ etc., describing (and/or parameterizing) the nontrivial structure of the QCD vacuum state. Formally, these terms appear from the operator product expansion for the amplitude $T(p_1^2, p_2^2, q^2)$ (fig.4):

$$T(p_1^2, p_2^2, q^2) = T^{\text{pert}}(p_1^2, p_2^2, q^2) + a \frac{\langle GG \rangle}{(p^2)^3} + b \frac{\alpha_s \langle \bar{q}q \rangle^2}{(p^2)^4} + \dots \quad (14)$$

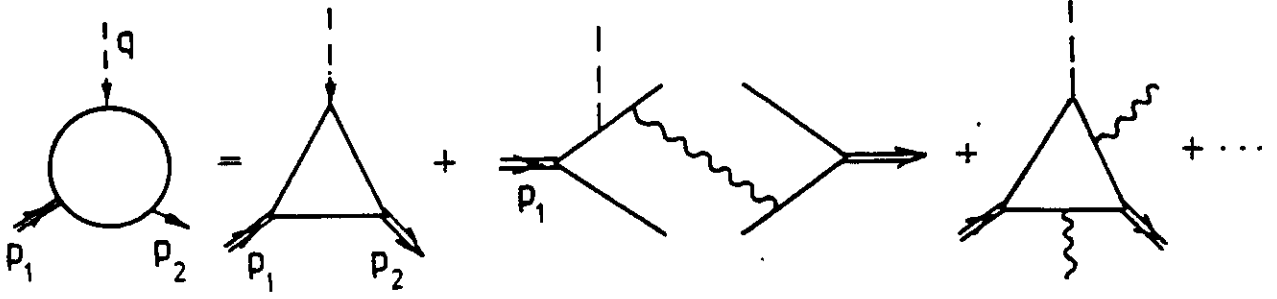


Fig.4 Structure of the operator product expansion for $T(p_1^2, p_2^2, q^2)$

The problem now is to construct such a model of the spectral density $\rho(s_1, s_2, q^2)$ which gives the best agreement between two expressions for T (eqs. (12) and (14)). Naturally, having only a few first terms of the $1/p^2$ -expansion one can hope to reproduce only the gross features of the hadronic spectrum in the relevant channel. Still, just using the simple fact that the condensate contributions die out for large p^2 , one obtains the *global duality* relation between quark and hadronic densities

$$\int_0^\infty ds_1 \int_0^\infty ds_2 (\rho(s_1, s_2, q^2) - \rho^{\text{pert}}(s_1, s_2, q^2)) = 0. \quad (15)$$

Approximating the higher states contribution into $\rho(s_1, s_2, q^2)$ by the free quark density (cf. with the quasiclassical approximation for high levels in quantum mechanics):

$$\rho^{\text{higher states}}(s_1, s_2, q^2) = [1 - \theta(s_1 < s_0)\theta(s_2 < s_0)] \rho^{\text{pert}}(s_1, s_2, q^2), \quad (16)$$

with s_0 being the effective threshold for the higher states production, one obtains the *local duality* relation

$$f_\pi^2 F_\pi(Q^2) = \frac{1}{\pi^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \rho^{\text{pert}}(s_1, s_2, q^2). \quad (17)$$

A similar duality relation can be obtained from the analysis of the correlator of two axial currents. In this case [10] $\rho(s) = \theta(s)/4\pi$ (see ref.[10]) and, hence, $f_\pi^2 = s_0/4\pi^2$. Thus, the duality interval s_0 for the pion can be fixed from the known experimental value of f_π :

$$s_0 = 4\pi^2 f_\pi^2 \approx 0.7 \text{ GeV}^2. \quad (18)$$

A more theoretical way is to use the standard values for the condensates [10] and extract both s_0 and f_π from the requirement of the best agreement between two expressions for the two-point analog of T . The result $f_\pi = 130 \pm 10 \text{ MeV}$ [10] is very close to the experimental value. In the same way, assuming (16) one can extract $f_\pi^2 F_\pi(Q^2)$ and s_0 from eqs. (12), (14) and (13). The fact that the values of s_0 obtained in this way are close to 0.7 GeV^2 [21, 22] is an evidence for the self-consistency of the whole approach.

Now, using the local duality relation (17), one can calculate $F_\pi(Q^2)$ without any free parameter [22]

$$F_\pi^{LD}(Q^2) = 1 - \frac{1 + 6s_0/Q^2}{(1 + 4s_0/Q^2)^{3/2}}. \quad (19)$$

Note, that asymptotically one has

$$F_{\pi}^{LD}(Q^2) = \frac{6s_0^2}{Q^4} + O(1/Q^6).$$

Comparing this with the perturbative QCD asymptotic contribution

$$F_{\pi}^{as}(Q^2) = \frac{8\pi\alpha_s f_{\pi}^2}{Q^2} = \left(\frac{\alpha_s}{\pi}\right) \frac{2s_0}{Q^2}, \quad (20)$$

one can establish that the power damping factor for $F_{\pi}^{LD}(Q^2)$ is

$$3s_0/Q^2 = 12\pi^2 f_{\pi}^2/Q^2 \approx 2GeV^2/Q^2,$$

in agreement with our expectations that the relevant scale M should be of the order of $1 GeV$. Though $F_{\pi}^{soft}(Q^2)$ behaves asymptotically like $1/Q^4$, in the accessible momentum transfer region eq. (19) is in reasonable agreement with experimental data (see fig.5, solid line). Furthermore, one can take into account in $\rho^{pert}(s_1, s_2, q^2)$ the $O(\alpha_s)$ contribution containing the asymptotically dominant hard gluon exchange term. The two-loop calculation is rather complicated, but one can use a simple model based on the interpolation

$$\frac{1}{\pi^2 f_{\pi}^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \Delta \rho^{pert}(s_1, s_2, q^2) \approx \frac{\alpha_s}{\pi} \frac{1}{1 + Q^2/2s_0} \quad (21)$$

between the $Q^2 = 0$ value (related by the Ward identity to the $O(\alpha_s)$ term of the 2-point correlator) and the asymptotic behavior. The resulting curve for $F_{\pi}(Q^2)$ is shown in fig.5 (dashed line).

Using the local quark-hadron duality relation of eq.(17) type, one can calculate also the nucleon form factors. In particular, for the proton magnetic form factor the result is [23]

$$G_M^p(Q^2) = \frac{8}{3} \sqrt{T^2 - 1} \left\{ (4T^2 - 1)(T^2 - 1) + (4T^2 - 3)T \sqrt{T^2 - 1} \right\}^{-1}, \quad (22)$$

where $T = 1 + Q^2/2s_0$ and the duality interval s_0 for the nucleon is $2.3 GeV^2$ [24]. Comparison with experimental data of ref. [25] is shown in fig.6. The theoretical curve starts to deviate from the data just in the region $Q^2 > 15 GeV^2$ where the $O(\alpha_s/\pi)$ -contribution should be visible. The latter is still not asymptotically dominant. The asymptotic term due to double-gluon-exchange diagrams is suppressed by the $O((\alpha_s/\pi)^2)$ factor and should be taken into account only for much higher Q^2 .

The QCD sum rule method can be used even to calculate the hadronic form factors in the low- Q^2 region, though the operator product expansion for $T(p_1^2, p_2^2, q^2)$ in this case is more complicated [26]: there appear new nonperturbative terms related to the long-distance propagation of quarks in the q -channel. These terms have the structure of a two-point correlator and can be calculated from the relevant QCD sum rule. In this way the result $\langle r_{\pi}^2 \rangle^{1/2} = 0.66 \pm 0.03 fm$ for the pion charge radius was obtained [27] in good agreement with the experimental value. The low- Q^2 QCD sum rule for the pion form factor [27] works up to $Q^2 \sim m_{\rho}^2 = 0.6 GeV^2$ and matches well with the curves obtained from the analysis of the QCD sum rule for the intermediate region $0.5 GeV^2 < Q^2 < 3 GeV^2$ performed in refs.[21, 22].

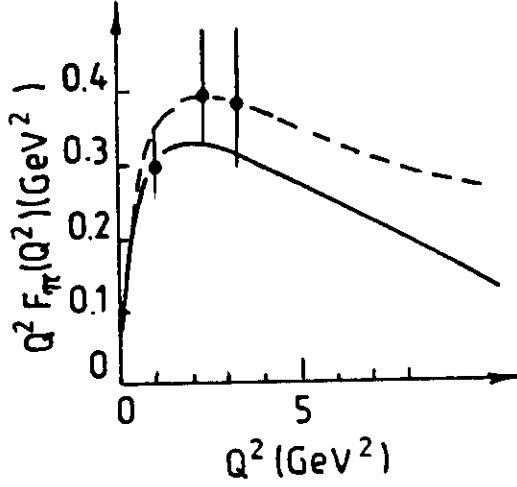


Fig.5 Pion form factor.

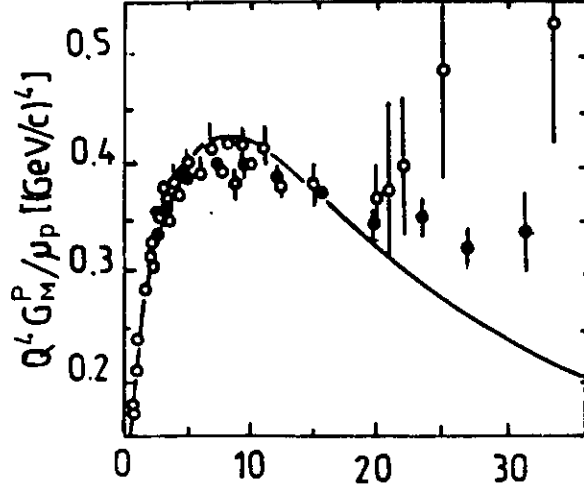


Fig.6 Proton magnetic form factor.

4. NONLOCAL CONDENSATES AND QCD SUM RULES FOR THE PION FORM FACTOR AND PION WAVE FUNCTION

The above-mentioned QCD sum rule for the pion form factor at intermediate momentum transfers

$$f_\pi^2 F_\pi(Q^2) = \frac{1}{\pi^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \rho^{pert}(s_1, s_2, q^2) \exp\left(-\frac{s_1 + s_2}{M^2}\right) + \frac{\alpha_s \langle GG \rangle}{12\pi M^2} + \frac{16\pi\alpha_s \langle \bar{q}q \rangle^2}{81 M^4} \left(13 + \frac{2Q^2}{M^2}\right) \quad (23)$$

(this is its "Borelized" version [21, 22], with $p^2 \rightarrow M^2$) cannot be used for $Q^2 > 3 \text{ GeV}^2$. This is because the ratio of the condensate terms (which do not decrease with the growth of Q^2) to the perturbative term (going down as $1/Q^4$) becomes too large, i.e., the power series in $1/M^2$ "explodes", and one should sum up it in some way.

A similar problem arises if one studies the QCD sum rule for the ξ -moments ($\xi = 2x - 1$) of the pion wave function [6]

$$f_\pi^2 \langle \xi^N \rangle = \frac{3M^2}{4\pi^2} \frac{1}{(N+1)(N+3)} (1 - e^{-s_0/M^2}) + \frac{\alpha_s \langle GG \rangle}{12\pi M^2} + \frac{16\pi\alpha_s \langle \bar{q}q \rangle^2}{81 M^4} (11 + 4N). \quad (24)$$

Here the perturbative term decreases like $1/N^2$ for higher moments while the condensate terms are either constant or even increasing with N . Thus, the effective scale in the channel (settled by the ratio of the condensate terms to the perturbative one) substantially increases for $N = 2, 4, \dots$. As a result, the value of the combination $f_\pi^2 \langle \xi^2 \rangle$ extracted from this sum rule is by

factor 2 larger than that for the asymptotic wave function: $f_\pi^2 \langle \xi^2 \rangle^{as} = f_\pi^2/5$. Such a large value $\langle \xi^2 \rangle \approx 0.4$ can be attributed only to a wave function concentrated in the region $|\xi| \sim 1$, *e.g.*, for the Chernyak-Zhitnitsky wave function (10) one has $\langle \xi^2 \rangle = 3/7 \sim 0.43$.

It is instructive to rewrite the sum rule (24) for the wave function itself:

$$f_\pi^2 \varphi_\pi(x) = \frac{3M^2}{2\pi^2} x(1-x)(1 - e^{-s_0/M^2}) + \frac{\alpha_s \langle GG \rangle}{24\pi M^2} [\delta(x) + \delta(1-x)] + \frac{8}{81} \frac{\pi \alpha_s \langle \bar{q}q \rangle^2}{M^4} \{11[\delta(x) + \delta(1-x)] + 2[\delta'(x) + \delta'(1-x)]\}. \quad (25)$$

The δ -functions in eq.(25) just indicate that the vacuum quarks carry zero fraction of the external momentum p . In the configuration representation this means that the nonlocal combination $\langle \bar{q}(z)q(0) \rangle$ is approximated by the local condensate $\langle \bar{q}(0)q(0) \rangle$, *i.e.*, vacuum fluctuations are assumed to have infinite correlation length. Such an approximation, though reasonable in many cases, is not always good enough. In particular, it was shown [28] that higher moments ($N = 2, 4, \dots$) of the pion wave function are rather sensitive to the width of the function $\langle \bar{q}(z)q(0) \rangle$, *i.e.*, to the average correlation length of the vacuum fluctuations, specified by the ratio

$$\lambda^2 \equiv \langle \bar{q} D^2 q \rangle / \langle \bar{q} q \rangle. \quad (26)$$

The numerical value of the parameter λ^2 , having the meaning of the average virtuality of the vacuum quarks is rather reliably determined from the QCD sum rules for the nucleon mass [24]

$$\langle \bar{q} D^2 q \rangle = \langle \bar{q}(\sigma \cdot G)q \rangle / 2 \approx \langle \bar{q} q \rangle (0.4 \text{ GeV}^2). \quad (27)$$

Note, that λ^2 is comparable in magnitude to the parameter $s_0 \approx 0.7 \text{ GeV}^2$, the latter being the characteristic scale in the pion channel. Using for $\langle \bar{q}(z)q(0) \rangle$ a model satisfying the above constraint *e.g.*, assuming the Gaussian form

$$\langle \bar{q}(z)q(0) \rangle = \langle \bar{q} q \rangle \exp(z^2 \lambda^2 / 8), \quad (28)$$

one can obtain a modified sum rule for $\varphi(x)$, with the $\delta(x)$ and $\delta(1-x)$ -functions substituted by terms like

$$x \theta(x < \lambda^2 / 2M^2) / \lambda^4 + \{x \rightarrow (1-x)\}.$$

For x small (or close to 1) the latter have the same behavior as the perturbative contribution. The value $\langle \xi^2 \rangle = 0.25$ obtained from that sum rule [28] is much closer to the asymptotic value $\langle \xi^2 \rangle^{as} = 0.2$ than to that of Chernyak and Zhitnitsky $\langle \xi^2 \rangle^{CZ} = 0.43$. The simplest model wave function with $\langle \xi^2 \rangle = 0.25$ is

$$\varphi_\pi^{mod,1}(x) = \frac{8}{\pi} f_\pi \sqrt{x(1-x)}. \quad (29)$$

If one prefers the expansion over the Gegenbauer polynomials $C_n^{3/2}(\xi)$ (the eigenfunctions of the evolution equation [2, 4]), then the function with $\langle \xi^2 \rangle = 0.25$ is

$$\varphi_\pi^{mod,2}(x) = 6f_\pi x(1-x) \left(1 + \frac{8}{9}(1 - 5x(1-x)) \right). \quad (30)$$

Similar changes appear also in the QCD sum rule for the pion form factor: if one takes into account the finite size of the vacuum fluctuations, then the condensate terms are decreasing functions of Q^2 , and the resulting sum rule can be used up to $Q^2 \sim 10 \text{ GeV}^2$. The curve for $F_\pi(Q^2)$ obtained in this way [29] goes slightly higher than the local duality prediction, eq.(11).

5. CONCLUSIONS

Let us now summarize the basic observations made above. First, we established that the gluons taking part in the "hard" rescattering subprocesses have normally rather low momenta $|k| < 500 \text{ MeV}$, i.e., they are outside the asymptotic freedom region. This has two implications: (a) $\alpha_s(k^2)$ could be large, and (b) one cannot be sure that the free-field approximation for the gluonic propagator $D(k^2) \sim 1/k^2$ is reliable.

Advocates of the asymptotic formulas argue that α_s is, in fact, small due to the freezing phenomenon [7, 14, 15]. We agree, that such a phenomenon should have place, simply because the propagation of the gluons in the low- k^2 region is strongly suppressed due to the confinement effects. However, modifying the gluonic (and quark) propagators $1/k^2 \rightarrow 1/(k^2 - M^2)$ in loops corresponding to radiative corrections, one should do the same for the original gluon of the "hard" subprocess. Such a modification substantially decreases the results obtained from the asymptotic formulas. In this situation the use of the wave functions *à la* Chernyak-Zhitnitsky is also of no help, because essential $|k^2|$'s in this case are even smaller and the suppression due to the $1/k^2 \rightarrow 1/(k^2 - M^2)$ change is much stronger.

Furthermore, the humpy form characteristic to the CZ-type wave functions is a mere consequence of the approximation that vacuum quarks have zero momentum. This approximation is good enough when one calculates f_π , i.e., the integrated wave function, but if one wants to get information concerning its form, one should know the momentum distribution of the vacuum quarks. In a realistic model of the QCD vacuum, with $\langle |k^2| \rangle^{\text{vac.}} \sim 0.4 \text{ GeV}^2$, the wave functions extracted from the QCD sum rules are very close to the asymptotic ones. This statement agrees with the lattice result [30] showing no skewness in the proton wave function.

All this forces us to conclude that the gluon exchange contributions cannot describe existing data in a theoretically convincing way, both in the pion and proton form factor cases.

Next step is to apply QCD sum rules for calculating soft contributions. This approach gives a self-consistent theoretical picture for the pion form factor behaviour starting from $Q^2 = 0$ up to $Q^2 \sim 10 \text{ GeV}^2$, and the soft contribution dominates in the whole region. A good description of the experimental data was obtained also for the proton form factor.

For more complicated processes like $\gamma\gamma \rightarrow \pi\pi$, $\gamma\gamma \rightarrow \bar{p}p$, $\pi p \rightarrow \pi p$, $pp \rightarrow pp$, etc., the asymptotically leading perturbative QCD contributions are suppressed by high powers of (α_s/π) , e.g., by $(\alpha_s/\pi)^4$ for πp -scattering and by $(\alpha_s/\pi)^5$ for pp -scattering and the deuteron form factor. However, reliable QCD calculations in these cases are much more complicated. Only for the deuteron form factor there was an attempt to calculate it using the QCD quark-hadron duality prescription [31], though much more work still should be done here.

Thus, after more than ten years of theoretical investigations, we have a reliable QCD description for only two simplest exclusive processes ($e\pi \rightarrow e\pi$ and $ep \rightarrow ep$). This is a pessimistic version of the statement. An optimist, however, can assert that from the analysis

of the pion and proton form factors we now know how the pion and the proton behave in exclusive reactions, and this gives us a reliable starting point and a correct direction towards a complete QCD theory of exclusive processes in general and hadronic form factors in particular.

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